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# Energy loss by electric and magnetic charges in matter

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**Abstract.** Exact expressions for the electromagnetic fields of a charged particle through a dispersive and dissipative medium have been used to calculate exact expressions for the energy loss due to distant collisions. These are evaluated using Fermi's one-oscillator approximation to the dielectric constant. For particle velocities with  $\beta < \epsilon_0^{-1/2}$  the results are identical to those of Fermi for an electric charge and to those of Ahlen for a magnetic monopole. For  $\beta > \epsilon_0^{-1/2}$  the present results differ from those of these authors. It is shown that this is due to their incomplete treatment of the conditions imposed by the model for the dielectric constant. The relevance of these results to the detection of magnetic monopoles and of dyons is also discussed.

## 1. Introduction

The concept of the magnetic charge as the magnetic counterpart to the electric charge has been admitted and utilised in classical physics by JJ Thomson (Adawi 1976, 1977). Its introduction within the framework of quantum mechanics is due to Dirac (1931), who utilised it in a fundamental manner so as to account for the quantisation of electric charge. Schwinger (1969) made the ultimate generalisation and postulated the existence of particles which carry both electric and magnetic charges. He called such particles dyons and he suggested their use so as to account for quark confinement.

These considerations were further reinforced through subsequent developments in gauge field theories. Thus 't Hooft (1974) and Polyakov (1974) independently succeeded in showing that such theories give rise to magnetic monopoles. Then Julia and Zee (1974) showed that such theories do also contain dyons among their possible solutions. As a consequence to these results, theoretical interest in magnetic monopoles, which has been strong from the very beginning (Recami 1978), was greatly enhanced (Goddard and Olive 1978, Craigie *et al* 1982).

The fundamental character of these concepts did not escape the attention of experimentalists. Thus as early as 1951 Malkus published the results of an experiment designed to detect monopoles produced in the atmosphere by primary cosmic rays. Since that time there has been a surge in experiments which searched for monopoles produced either by cosmic rays or by accelerator beams, or trapped in terrestrial, meteoric or lunar rocks. But by the time of the Jones (1977) survey only one experiment (Price *et al* 1975) reported a positive result consisting of the detection of a magnetic monopole in the primary cosmic ray flux. The result of this experiment generated considerable interest at the time and led to subsequent comments by other investigators. Fleischer and Walker (1975) maintained that the data could not preclude alternative interpretations, and Friedlander (1975) held that this was rather likely.

This prompted Price *et al* (1978) to undertake further analysis of their experiment which led them to exclude completely the possibility of having detected a magnetic monopole.

Since then, Hoffman *et al* (1978) reported the negative result of a search at the CERN-ISR, and Bartlett *et al* (1978) reported the negative findings of a search in cosmic rays. In a very recent survey of the subject, Giacomelli (1982) concludes that all experimental searches so far have been negative. However, more recently Cabrera (1982) published first results of an experiment in which he reports detecting a magnetic monopole through its passage within a superconducting ring.

This uncertain situation and the impact of its outcome have prompted many calculations of the behaviour of charged particles in matter (Ahlen 1982, Ahlen and Kinoshita 1982, Ford 1982, Trefil 1982). This is very relevant to the experimental effort aimed at the detection of magnetic monopoles, since this relies on the contrast between their behaviour and that of electric charges. The present work must be conceived in this vein. It bears on experiments which utilise ionisation loss for the detection of magnetic monopoles. In what follows we will try to make a case for the use, in this respect, of exact expressions for the electromagnetic fields of a charged particle passing through a dispersive and dissipative medium which we have obtained in recent work (Saffouri 1982).

The first instance in which these fields would be of value is in the calculation of the so-called density effect in the energy loss by charged particles in dense media (Ahlen 1980). This effect, as Ahlen points out, is quite adequately treated in a classical fashion. Hence, our expressions for the fields are ideally suited for this calculation.

Second, these fields are the natural choice in the evaluation of the stopping power for magnetic monopoles in the manner carried out by Ahlen (1976, 1978).

Third, since these expressions for the fields take into account both electric and magnetic dispersion, they should be well suited to treatments similar to that carried out by Martem'yanov and Khakimov (1972).

To justify these claims, we calculate the density effect. We do this within the approximations utilised by Fermi in his original article (Fermi 1940). This should suffice to exhibit the usefulness and relevance of our results. The results of our calculations for electric charges agree with Fermi's calculation for the case of particle velocities smaller than the phase velocity of electromagnetic radiation in the medium. For particle velocities greater than this phase velocity, our result is markedly lower than that given by Fermi. This arises out of the circumstance that the fields acquire different forms in either velocity range and that the domain of affected frequencies is also different. It should be possible to observe this effect experimentally. But then a more realistic model for the medium should be used along the lines first followed by Halpern and Hall (1948) and by Sternheimer (1952) and subsequently perfected by Sternheimer (1956, 1966, 1967).

For magnetic monopoles we compare our results with those given by Ahlen, which we have referred to above. This is quite significant since we apply the same approximations which he introduces. We also use our calculations to give an indication of the behaviour of dyons. By going to the asymptotic limit of the fields, we suggest a method for the detection of dyons which utilises the Cerenkov radiation emitted by them. This method has the advantage that it could, in principle, determine their relative charge.

The arrangement of material is as follows. In § 2 we use the exact electromagnetic fields for either an electric or a magnetic charge moving in a dispersive and dissipative

medium to derive exact expressions for the energy loss by the particle. In § 3 we discuss the model of the dielectric constant which we are using and the restrictions which it imposes on our calculation. In particular, we give a careful critique of Fermi's calculation and we point out the limitations which make it differ from ours. In § 4 we use the approximation procedure outlined in § 3 to evaluate the energy loss expressions. In § 5 we compare our results with those of Fermi and of Ahlen, and in § 6 we sketch the relevance of our results to the detection of the dyon. We present our conclusions in § 7.

## 2. The expression for the energy loss

We consider a material medium which exhibits both dispersion and dissipation to electromagnetic radiation. Its dielectric constant,  $\epsilon$ , and magnetic permeability,  $\mu$ , will then be complex functions of the frequency  $\omega$  of the radiation and will satisfy the following two relations:

$$\epsilon(\omega) = \epsilon^*(-\omega), \quad \mu(\omega) = \mu^*(-\omega). \quad (2.1)$$

In the above the asterisk stands for complex conjugation. A particle carrying either magnetic or electric charge will lose energy upon passage through the medium due to its dissipative nature. When the velocity of the particle exceeds that of the phase velocity of electromagnetic radiation in the medium, it will also emit Cerenkov radiation.

The density effect arises out of the circumstance that for the distant collisions between the charged particle and the atoms of the medium, the polarisation of the medium must be taken into account. These collisions are defined by a minimum impact parameter of the order of atomic dimensions (Ahlen 1980). The respective energy loss is calculated via the standard methods of classical electrodynamics (Jackson 1975).

Thus for a particle moving with velocity  $v$  along the  $z$  axis the energy loss is calculated via the radial component of the Poynting flux. Due to the cylindrical symmetry of the problem, this will be given by a function of the form  $S(\rho, z, t)$ , where  $(\rho, z)$  are cylindrical coordinates and  $t$  is the time. The energy loss by the particle per unit of path length per unit time is given by

$$d^2U/dz dt = 2\pi\rho S(\rho, z, t).$$

In Saffouri (1982) we showed that the energy loss per unit of path length is independent of  $z$  and is given by the expression

$$\frac{dU}{dz} = 2\pi\rho \int_{-\infty}^{\infty} S(\rho, z, t) dt. \quad (2.2)$$

This expression gives the energy which the particle deposits per unit of path length in the region of space starting at a distance  $\rho$  away from its path. In particular, in the case when the particle emits Cerenkov radiation, this formula will include energy lost in this form also.

Equation (2.2) is our starting point for the calculation of the energy loss. We must then evaluate the Poynting flux. For this we need the electromagnetic fields of the particle, which we take from Saffouri (1982). We have shown there that these fields will have two forms depending on whether the velocity of the particle is smaller or

greater, than the phase velocity of electromagnetic radiation in the medium. We have used the symbol  $c'(\omega)$  to represent the phase velocity of electromagnetic radiation of frequency  $\omega$ , and we showed that

$$c'(\omega) = c/(\text{Re } \epsilon(\omega)\mu(\omega))^{1/2}. \quad (2.3)$$

In the above,  $c$  is the velocity of light in vacuum and  $\text{Re}$  stands for the real part of the expression following it. Introducing the symbol  $\kappa^2(\omega)$ ,

$$\kappa^2(\omega) \equiv \epsilon(\omega)\mu(\omega) \quad (2.4)$$

we can then characterise the above two conditions by  $\text{Re } \kappa^2(\omega) < 1/\beta^2$  and  $\text{Re } \kappa^2(\omega) > 1/\beta^2$  respectively, where  $\beta = v/c$ . We now turn to the calculation of the radiation loss, where we first treat the case of the magnetic charge and then that of an electric charge.

### 2.1. Energy loss for a magnetic charge

We consider a particle which carries magnetic charge  $e'$  and which moves with velocity  $v$  along the  $z$  axis. Our treatment must then distinguish between the two regions of values of  $v$  which we have just mentioned.

*Case 1.*  $v < c'(\omega)$ . The fields are given by

$$\begin{aligned} \mathbf{E}(\rho, z, t) &= \varphi_1 \frac{e'}{\pi\beta c^2} \left[ \int_{\text{Re}\kappa^2(\omega) < 1/\beta^2} d\omega \frac{\omega}{\gamma'(\omega)} e^{i\omega(z/v-t)} K_1\left(\frac{\omega\rho}{\gamma'v}\right) + \text{CC} \right], \\ \mathbf{B}(\rho, z, t) &= \frac{e'}{\pi\beta^2 c^2} \left[ \int_{\text{Re}\kappa^2(\omega) < 1/\beta^2} d\omega \frac{\omega}{\gamma'(\omega)} e^{i\omega(z/v-t)} \right. \\ &\quad \times \left( \mathbf{k} \frac{iK_0(\omega\rho/\gamma'v)}{\gamma'(\omega)} - \rho_1 K_1\left(\frac{\omega\rho}{\gamma'v}\right) \right) + \text{CC} \left. \right], \end{aligned} \quad (2.5)$$

where

$$\gamma'(\omega) = (1 - \beta^2 \kappa^2(\omega))^{-1/2}. \quad (2.6)$$

$K_0$  and  $K_1$  are the modified Bessel functions of the third kind of order zero and one, respectively,  $(\rho_1, \phi_1, \mathbf{k})$  are the cylindrical unit vectors, and CC stands for the complex conjugate.

We use these expressions for the fields in order to calculate the Poynting flux. We substitute this in (2.2) to obtain

$$\frac{dV}{dz} = \frac{2}{\pi} \frac{e'^2 \rho}{\beta^3 c^3} \text{Re} \int_{\text{Re}\kappa^2(\omega) < 1/\beta^2} d\omega \frac{i\omega^2}{\mu(\omega)|\gamma'(\omega)|^2 \gamma'(\omega)} K_0\left(\frac{\omega\rho}{\gamma'v}\right) K_1\left(\frac{\omega\rho}{\gamma'^*v}\right). \quad (2.7)$$

This is the exact expression for the energy deposited per unit of its path length by the magnetic charge in the region of space beyond the radial distance  $\rho$ .

*Case 2.*  $v < c'(\omega)$ . The fields now have the following form:

$$\mathbf{E}(\rho, z, t) = \varphi_1 \frac{e'}{\pi\beta c^2} \left[ \int_{\text{Re}\kappa^2(\omega) > 1/\beta^2} d\omega \frac{i\omega}{\gamma'(\omega)} e^{i\omega(z/v-t)} H_1^{(1)}\left(\frac{\omega\rho}{\gamma'v}\right) + \text{CC} \right],$$

$$\mathbf{B}(\rho, z, t) = \frac{e'}{2\beta^2 c^2} \left\{ \int_{\text{Re}\kappa^2(\omega) > 1/\beta^2} d\omega \frac{\omega}{\gamma'(\omega)} e^{i\omega(z/v-t)} \times \left[ \left( k \frac{H_0^{(v)}(\omega\rho/\gamma'v)}{\gamma'(\omega)} - \rho_1 i H_1^{(1)}\left(\frac{\omega\rho}{\gamma'v}\right) \right) + \text{cc} \right] \right\}, \tag{2.8}$$

where  $H_0^{(1)}$  and  $H_1^{(1)}$  are Hankel functions of the first kind of order zero and one, respectively, and  $\gamma'(\omega)$  is now defined as follows:

$$\gamma'(\omega) = (\beta^2 \kappa^2(\omega) - 1)^{-1/2}. \tag{2.9}$$

The corresponding energy loss expression is given by

$$\frac{dU}{dz} = \frac{\pi}{2} \frac{e'^2 \rho}{(\beta c)^3} \text{Re} \int_{\text{Re}\kappa^2(\omega) > 1/\beta^2} d\omega \frac{-i\omega^2}{\mu(\omega) |\gamma'(\omega)|^2 \gamma'(\omega)} H_0^{(1)}\left(\frac{\omega\rho}{\gamma'v}\right) H_1^{(1)}\left(\frac{\omega\rho}{\gamma'^*v}\right). \tag{2.10}$$

### 2.2. Energy loss for an electric charge

The fields for the electric charge are obtained from those which we have given above for the magnetic charge via the following standard substitution. Hence, we do not need to exhibit them explicitly here. We discuss the fields in the following obvious notation:

$$\mathbf{E}_{\text{electric}} = e \int d\omega \mathbf{E}_{\text{electric}}(\omega).$$

The substitution is then as follows:

$$\mathbf{E}_{\text{electric}} = -e \int d\omega \frac{\mathbf{B}_{\text{mag}}(\omega)}{\varepsilon(\omega)}, \quad \mathbf{B}_{\text{electric}} = e \int d\omega \mu(\omega) \mathbf{E}_{\text{mag}}(\omega). \tag{2.11}$$

Using the resulting fields in (2.2), we obtain the following expression for the energy loss for the case  $v < c'(\omega)$ :

$$\frac{dU}{dz} = \frac{2}{\pi} \frac{e'^2 \rho}{\beta^3 c^3} \text{Re} \int_{\text{Re}\kappa^2(\omega) < 1/\beta^2} d\omega \frac{i\omega^2}{\varepsilon(\omega) |\gamma'(\omega)|^2 \gamma'(\omega)} K_0\left(\frac{\omega\rho}{\gamma'v}\right) K_1\left(\frac{\omega\rho}{\gamma'^*v}\right) \tag{2.12}$$

where

$$\gamma'(\omega) = (1 - \beta^2 \kappa^2(\omega))^{-1/2}.$$

This is the main result given by equation (2.2) in Fermi (1940).

The corresponding result for the case  $v > c'(\omega)$  is now given by

$$\frac{dU}{dz} = \frac{\pi}{2} \frac{e'^2 \rho}{\beta^3 c^3} \text{Re} \int_{\text{Re}\kappa^2(\omega) > 1/\beta^2} d\omega \frac{-i\omega^2}{\varepsilon(\omega) |\gamma'(\omega)|^2 \gamma'(\omega)} H_0^{(1)}\left(\frac{\omega\rho}{\gamma'v}\right) H_1^{(1)}\left(\frac{\omega\rho}{\gamma'^*v}\right) \tag{2.13}$$

where

$$\gamma'(\omega) = (\beta^2 \kappa^2(\omega) - 1)^{-1/2}.$$

### 2.3. Comments

A few remarks are in order here. First, we see from the preceding results that the energy loss for a magnetic charge is obtained from that for an electric charge by

replacing  $(e^2/\epsilon(\omega))$  in the spectral integral by  $(e'^2/\mu(\omega))$ . This is a rigorous demonstration of a well known rule (Ahlen 1976). Furthermore, since for dielectric media, for instance,  $\mu(\omega) = 1$ , while  $\epsilon(\omega)$  is highly frequency-dependent, this replacement will have as a consequence that the energy loss for monopoles will be quite distinct from that for electric charges.

Second, the above four expressions for the energy loss are all exact. By putting into them the appropriate expressions for the susceptibilities of the medium, we can, in principle, calculate the correction due to the density effect to any desired accuracy.

Third, we note that the results for  $v < c'(\omega)$  can be obtained from those for  $v > c'(\omega)$  and *vice versa* via the following substitution:

$$(\beta^2 \kappa^2 - 1)^{-1/2} \leftrightarrow i^{-1} (1 - \beta^2 \kappa^2)^{-1/2}.$$

However, we must point out that in the actual evaluation of the preceding expressions for a specific model for the dielectric constant of the medium, we cannot simply obtain one result from the other through this strategem. The hindrance comes from the limits of integration which are not the same in both cases. We will clarify this matter further in § 3.

### 3. General considerations

In evaluating the above expressions, we will follow Fermi and make two major assumptions. The first concerns the medium and the second concerns the magnitude of the distance  $\rho$  beyond which the energy is deposited. As to the medium we will assume it to be a pure dielectric. This allows us to put the magnetic permeability equal to unity:

$$\mu = 1. \tag{3.1}$$

Furthermore, we will assume that the dielectric constant of the medium is given by the single oscillator formula. Adopting Fermi's notation, we write this as

$$\epsilon(\omega) = 1 + \frac{(4\pi n e^2/m)}{\omega_0^2 - \omega^2 - 2i p \omega} \tag{3.2}$$

where  $e$  and  $m$  are the charge and the mass of the electron,  $n$  is the number of electrons per unit volume,  $\omega_0$  is the natural frequency of the oscillator, and  $2p$  is the coefficient of the dissipative force. We introduce the electronic plasma frequency of the medium  $\omega_p$ :

$$\omega_p^2 = 4\pi n e^2/m \tag{3.3}$$

and the static dielectric constant  $\epsilon_0$ :

$$\epsilon_0 = \epsilon(0) = 1 + (\omega_p/\omega_0)^2. \tag{3.4}$$

Introducing the dimensionless variable  $x$

$$x = (\omega/\omega_0) \tag{3.5}$$

we express the dielectric constant in the following explicitly dimensionless form:

$$\epsilon(\omega) = \epsilon(x) = 1 + (\epsilon_0 - 1)/(1 - x^2 - 2i\eta x) \tag{3.6}$$

where

$$\eta = (P/\omega_0). \tag{3.7}$$

Whenever necessary we will assume that the dimensionless constant  $\eta$  is very small:

$$\eta \ll 1. \tag{3.8}$$

This implies that we are considering a medium with little dissipation.

As to the variable  $\rho$  we will fix its value by putting it equal to a distance  $b$  of the order of an angstrom:

$$\rho = b \approx 1 \text{ \AA}. \tag{3.9}$$

We will then assume that the resulting argument of the various forms of Bessel functions occurring above; namely, the variable  $\omega b / \gamma'(\omega) \beta c$  is so small as to allow us to use the small argument expansions for these functions.

### 3.1. The limits of integration

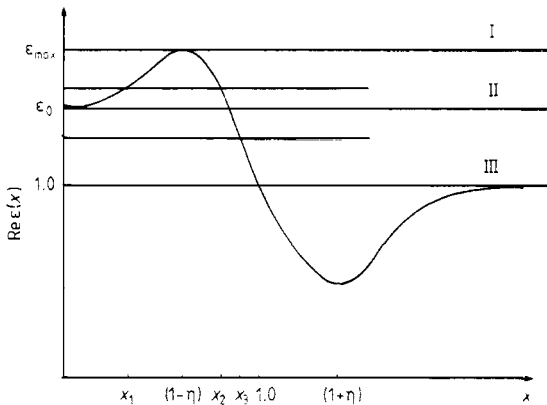
In order to evaluate the integrals which occur in the expressions for the energy loss, we must find the domain of allowed values of the frequency—and equivalently  $x$ —which is selected by either of the two inequalities restricting the range of values of  $\text{Re } \kappa^2(\omega)$ . Due to equation (3.1) the function  $\kappa^2(\omega)$  is now given by

$$\kappa^2(\omega) = \kappa^2(x) = \varepsilon(x).$$

Substituting from (3.6) into this relation, we obtain

$$\text{Re } \kappa^2(\omega) = \text{Re } \varepsilon(x) = 1 + (\varepsilon_0 - 1)(1 - x^2) / [(1 - x^2)^2 + (2\eta x)^2]. \tag{3.10}$$

We give a sketch of this familiar function in figure 1, where we already make use of the smallness of  $\eta$  to keep terms up to the first order in it. By marking off the values of  $1/\beta^2$  on the  $\text{Re } \varepsilon(x)$  axis we translate the inequalities into ranges of values of  $\text{Re } \kappa^2(x)$ .



**Figure 1.** A schematic plot of the function  $\text{Re } \varepsilon(x)$  against  $x$ . The three Roman numerals indicate the three regions of velocity discussed in the text.

To start with, since  $1/\beta^2 > 1$  all the possible values of  $1/\beta^2$  lie above the horizontal line at  $\varepsilon = 1$ . This range of values divides into three regions which give different domains of allowed values for  $x$ . We indicate these three regions on figure 1 and we give here their definition.



$$\begin{aligned}
 \text{I: } & \operatorname{Re} \kappa^2(x) > \varepsilon_{\max}, \\
 \text{II: } & < \varepsilon_0 < \operatorname{Re} \kappa^2(x) < \varepsilon_{\max}, \\
 \text{III: } & 1 < \operatorname{Re} \kappa^2(x) < \varepsilon_0
 \end{aligned}
 \tag{3.11}$$

where

$$\varepsilon_{\max} = [(\varepsilon_0 + 3)/4 + (\varepsilon_0 - 1)/4\eta]. \tag{3.12}$$

For each of the two velocity ranges the appropriate results will be as follows.

*Case 1.*  $v < c'(\omega)$ . Since now  $\operatorname{Re} \kappa^2(\omega) > 1/\beta^2$ , the value of  $1/\beta^2$  can lie in any of the three regions. For each region we list the resulting domain of integration over  $x$ :

$$\begin{aligned}
 \text{I: } & [0, \infty], \\
 \text{II: } & [0, x_1] \cup [x_2, \infty), \\
 \text{III: } & [x_3, \infty].
 \end{aligned}
 \tag{3.13}$$

In the above,  $x_1, x_2$  are the abscissae of the intercepts of a general value of  $1/\beta^2$  lying in region II with the curve  $\operatorname{Re} \kappa^2(x)$  as seen in figure 1. The same is true for  $x_3$  but in region III. The symbol  $\cup$  stands for union.

*Case 2.*  $v > c'(\omega)$ . In this case  $\operatorname{Re} \kappa^2(\omega) > 1/\beta^2$ , and the value of  $1/\beta^2$  can lie only in regions II and III. For these the allowed domains of integration are as follows:

$$\text{II: } [x_1, x_2], \quad \text{III: } [0, x_3], \tag{3.14}$$

where  $x_1, x_2, x_3$  have the same meaning as in the preceding case.

### 3.2. Comments on Fermi's calculation.

We are now in a position to clarify the remarks which we made in § 2 concerning Fermi's treatment of the energy loss by electric charges. Fermi considers two ranges of particle velocities which he distinguishes via the sign of the following parameter:

$$a = (1 - \varepsilon_0 \beta^2)/(1 - \beta^2). \tag{3.15}$$

Thus for  $a > 0$ , we have  $1/\beta^2 < \varepsilon_0$ , which means that  $1/\beta^2$  lies either in region I or in region II of figure 1. For this case Fermi uses equation (1.12) to evaluate the energy loss and chooses  $[0, \infty]$  as the domain of integration. In view of our relations (3.11) and (3.13) this means that his calculation in this case holds for particle velocities which satisfy the condition

$$1/\beta^2 > \varepsilon_{\max}.$$

In other words, they are velocities which lie in region I of figure 1. Using equation (3.12) for  $\varepsilon_{\max}$  and keeping terms up to lowest order in  $\eta$ , we can write the above relation in the form

$$\beta^2 \ll \{[(\varepsilon_0 - 1)/4\eta] + 1\}^{-1} \approx 0.04. \tag{3.16}$$

We will give the justification for this result in a later section. For the time being we remark that this region includes the range of velocities from relatively appreciable values down to the order of atomic velocities:  $\beta \sim \alpha$  (Ahlen and Kinoshita 1982).

The alternative case corresponding to  $a < 0$  is characterised by  $1/\beta^2 < \epsilon_0$ . This means that  $1/\beta^2$  lies in region III of figure 1. The velocity would lie either in the range  $v < c'(\omega)$  or in the complementary range  $v > c'(\omega)$ . For the first range we must use (2.12) in calculating the energy loss together with the domain of integration given by

$$[x_3, \infty),$$

where  $x_3$  satisfies the inequality

$$(1 - 2\eta^2) < x_3 < 1.$$

Since we take  $\eta$  to be very small, we see that

$$x_3 \approx 1.$$

For the second range of velocities we must use (2.13) with the domain of integration given by  $[0, x_3]$ .

Fermi calculates this case by evaluating (2.12) for  $a < 0$  and retaining the same domain of integration which he used in the preceding case; namely,  $[0, \infty]$ . This he takes to be the result of energy loss for  $v < c'(\omega)$ . Our discussion so far would seem to indicate that this procedure is inconsistent with our treatment.

In view of this we feel justified in presenting a consistent treatment of this case in what follows. As we shall see later our result will differ markedly from that given by Fermi for this case. This would call for a careful experimental examination to decide in favour of either treatment.

### 3.3. Choice of the limits of integration

*Case 1.  $v < c'(\omega)$ .* Since in this case there are three possible forms for these limits depending on which region of particle velocities we consider, a possible approach would be to calculate the energy loss for each region. This would indeed be the proper procedure to follow in a more exhaustive treatment employing a more realistic model for the medium in the manner utilised, for instance, by Sternheimer (1952, 1956, 1966, 1967). However, since we intend to use Fermi's result for electric charges in this range of velocities, we must adopt his choice of limits for our treatments of the corresponding case of magnetic charges. Consequently, we will start from (2.7) and use in it the domain of integration given by  $[0, \infty]$ .

*Case 2.  $v > c'(\omega)$ .* In this case the value  $1/\beta^2$  lies either in region II or in region III of figure 1. In the preceding case we have chosen the region which allows us to go to the limit of small velocities. To obtain maximum contrast with that case we must choose now the region which allows us to go to the opposite limit of high velocities. This means that we must choose  $1/\beta^2$  to lie in region III. To be specific, we will consider ranges of  $\beta^2$  which satisfy the following condition:

$$1/\epsilon_0 < \beta^2 \leq (1 - \eta). \tag{3.17}$$

We write the corresponding domain of integration over  $x$  in the form

$$[0, 1 - \delta] \tag{3.18}$$

since we expect  $\delta$  to be a small number. To find  $\delta$ , we put  $\text{Re } \epsilon(x) = 1/\beta^2$  and  $x = (1 - \delta)$  in (3.10) and we solve the resulting equation for  $\delta$ . To lowest order in  $\eta$ ,

the solution is given by the expression

$$\delta = 2\eta^2 / (\epsilon_0 - 1)\gamma^2 \tag{3.19}$$

where

$$\gamma^2 = (1 - \beta^2)^{-1/2}.$$

In the present case the upper limit of integration depends on the velocity and the above equation gives this dependence correct to lowest order in  $\eta$ .

**4. Evaluation of the energy loss expressions**

*4.1. Energy loss by a moving magnetic charge*

*Case 1.  $v < c'(\omega)$ .* We start with (2.7) and we use in it the limits of integration appropriate to the velocity range specified by (3.16). We also put  $\mu(\omega) = 1$  in accordance with (3.1) and we replace  $\rho$  by the constant  $b$  to obtain the relation

$$\frac{dU}{dz} = \frac{2}{\pi} \frac{e'^2 b}{\beta^3 c^3} \operatorname{Re} \int_0^\infty d\omega \frac{i\omega^2}{|\gamma'(\omega)|^2 \gamma'(\omega)} K_0\left(\frac{\omega b}{\gamma' v}\right) K_1\left(\frac{\omega b}{\gamma'^* v}\right). \tag{4.1}$$

Next we approximate the Bessel functions in the integrand above by their small argument values and we substitute the expression (3.6) for  $\epsilon(\omega)$ . Using the dimensionless variable  $x$ , defined by (3.5), we write the above relation as

$$\begin{aligned} \frac{dU}{dz} = & - \frac{4n(ee')^2}{(\epsilon_0 - 1)\beta^2 mc^2} \operatorname{Re} \int_0^\infty dx \left\{ ix \left( \frac{1}{\gamma^2} - \frac{\beta^2(\epsilon_0 - 1)}{(1 - x^2 - 2i\eta x)} \right) \right. \\ & \left. \times \left[ \log \left( \frac{3.17\pi ne^2 b^2}{(\epsilon_0 - 1)m\gamma^2 v^2} \right) + \log x^2 + \log \left( \frac{a - x^2 - 2i\eta x}{1 - x^2 - 2i\eta x} \right) \right] \right\} \end{aligned} \tag{4.2}$$

where  $a$  is defined by (3.15).

We have expressed our result in this form so as to correspond to the expressions given by Fermi (1940). The above integral splits up into six integrals which have all been evaluated by Fermi and we take them from his article. Our final result is then given as follows:

$$\frac{dU}{dz} = 2\pi \frac{n(ee')^2}{mc^2} \left[ \log \left( \frac{mc^2 \beta^2}{3.17\pi ne^2 b^2} \right) + \log \left( \frac{\epsilon_0 - 1}{1 - \beta^2} \right) - 1 - \frac{2\eta}{(1 - \eta^2)^{1/2}} \tan^{-1} \frac{(1 - \eta^2)^{1/2}}{\eta} \right]. \tag{4.3}$$

This is to be compared with the corresponding expression obtained for the case of an electric charge by Fermi:

$$\begin{aligned} \frac{dU}{dz} = & \frac{2\pi ne^4}{mc^2 \beta^2} \left[ \log \left( \frac{mc^2 \beta^2}{3.17\pi ne^2 b^2} \right) \right. \\ & \left. + \log \left( \frac{(\epsilon_0 - 1)}{\epsilon_0(1 - \beta^2)} \right) - \beta^2 - \frac{2\eta}{(1 - \eta^2)^{1/2}} \tan^{-1} \frac{(1 - \eta^2)^{1/2}}{\eta} \right]. \end{aligned} \tag{4.4}$$

We will find it useful at this stage to introduce the plasma wavelength  $\lambda_p$ :

$$\lambda_p = c/\omega_p. \tag{4.5}$$

With the aid of  $\lambda_p$  and  $\omega_p$  we define a characteristic energy density for the medium:

$$u_0 = \hbar\omega_p/\lambda_p. \tag{4.6}$$

Using these quantities, we express the above results in the form

$$\begin{aligned} \frac{dU}{dz} = \alpha' u_0 \left[ \frac{1}{2} \log \left( \frac{4}{3.17} \frac{\beta^2}{(b/\lambda_p)^2} \right) \right. \\ \left. + \frac{1}{2} \log[(\epsilon_0 - 1)\gamma^2] - \frac{1}{2} - \frac{\eta}{(1 - \eta^2)^{1/2}} \tan^{-1} \left( \frac{(1 - \eta^2)^{1/2}}{\eta} \right) \right]. \end{aligned} \tag{4.7}$$

$$\begin{aligned} \frac{dU}{dz} = \alpha u_0 \left[ \frac{1}{2\beta^2} \log \left( \frac{4}{3.17} \frac{\beta^2}{(b/\lambda_p)^2} \right) \right. \\ \left. + \frac{1}{2\beta^2} \log \left( \frac{\epsilon_0 - 1}{\epsilon_0} \gamma^2 \right) - \frac{1}{2} - \frac{\eta}{\beta^2(1 - \eta^2)^{1/2}} \tan^{-1} \left( \frac{(1 - \eta^2)^{1/2}}{\eta} \right) \right], \end{aligned} \tag{4.8}$$

where  $\alpha'$  is the magnetic fine structure constant:

$$\alpha' = e'^2/\hbar c \tag{4.9}$$

and  $\alpha$  is the usual fine structure constant. We will defer discussion of these results until § 5. However, we point out now that (4.7) is identical with the expression, correct to zero order in  $\eta$ , used by Ahlen (1976) for the respective velocity range. His result was originally derived by Tompkins (1965) starting from Fermi's result which we quote here.

*Case 2.*  $v > c'(\omega)$ . Our starting point now is expression (2.10) for the energy loss. We apply to it the same approximations which we used in the preceding case and we use in it the limits of integration given by (3.18) which are appropriate to the velocity range (3.17). The resulting expression has the form

$$\begin{aligned} \frac{dU}{dz} = \frac{4\pi n(ee')^2}{(\epsilon_0 - 1)\beta^2 m c^2} \operatorname{Re} \int_0^{(1-\delta)} dx x \left( \frac{\beta^2(\epsilon_0 - 1)}{(1 - x^2 - 2i\eta x)} - \frac{1}{\gamma^2} \right) \\ \times \left\{ 1 + \frac{i}{\pi} \left[ \log \left( \frac{3.17\pi n e^2 b^2}{(\epsilon_0 - 1)m\gamma^2 v^2} \right) + \log x^2 + \log \left( \frac{a + x^2 + 2i\eta x}{1 - x^2 - 2i\eta x} \right) \right] \right\} \end{aligned} \tag{4.10}$$

where

$$a = [(\epsilon_0\beta^2 - 1)/(1 - \beta^2)]. \tag{4.11}$$

This expression is quite distinct from that given by (4.2) for the velocity range  $v < c'(\omega)$ . This should further clarify our contention in § 3 that it is not permissible to use the same expression, albeit with the appropriate limitations on the parameter  $a$ , to drive the energy loss in either velocity range.

The finite domain of integration in the present case complicates the evaluation of the integrals as compared with the preceding case. However, we do not need to undertake the arduous task of trying to evaluate these integrals exactly—which indeed may not be possible for some of them. Our aim from the very beginning has been to obtain expressions in powers of the small parameter  $\eta$ . Hence, we will expand this integrand in powers of  $\eta$  and then keep terms up to the first order only. The result

is as follows:

$$\frac{dU}{dz} = \frac{4\pi n(ee')^2}{(\epsilon_0 - 1)\beta^2 mc^2} \int_0^{(1-\delta)} dx \left[ \left( \beta^2(\epsilon_0 - 1) \frac{x}{1-x^2} - \frac{x}{\gamma^2} \right) + \frac{2}{\pi} \eta \left\{ \frac{1}{\gamma^2} \left( \frac{x^2}{a+x^2} + \frac{x^2}{1-x^2} \right) \right. \right. \\ \left. \left. - \beta^2(\epsilon_0 - 1) \left[ \left( 1 + \log \frac{3.17\pi ne^2 b^2}{(\epsilon_0 - 1)m\gamma^2 v^2} \right) \left( \frac{x}{1-x^2} \right)^2 \right. \right. \right. \\ \left. \left. + \frac{x^2}{1-x^2} \frac{1}{a+x^2} + \frac{x^2 \log x^2}{(1-x^2)^2} + \frac{x^2}{(1-x^2)^2} \log \left( \frac{a+x^2}{1-x^2} \right) \right] \right\} \right] \quad (4.12)$$

The above integrals are tedious but they can all be evaluated in an elementary fashion. Only the last two terms need special attention. This warrants our including the result of evaluating them in an appendix. We give here the result of integration of the above expression:

$$\frac{dU}{dz} = \frac{4\pi n(ee')^2}{(\epsilon_0 - 1)\beta^2 mc^2} \left[ \left[ - \left( \frac{\beta^2(\epsilon_0 - 1)}{2} \log(1-x^2) + \frac{x^2}{2\gamma^2} \right) \right. \right. \\ \left. \left. - \frac{\beta^2(\epsilon_0 - 1)}{\pi} \eta \left\{ \frac{x}{(1-x^2)} \log \frac{\zeta x^2(a+x^2)}{(1-x^2)} + \frac{1}{2} \log(1-x) \log \left( \frac{1+x}{4} \right) \right. \right. \right. \\ \left. \left. - 2 \left[ \frac{a+2}{a+1} + \frac{1}{4} \log \left( \frac{\zeta(a+1)x^2}{(1-x^2)} \right) \right] \tanh^{-1} x \right. \right. \\ \left. \left. + \frac{2\sqrt{a}}{a+1} \tan^{-1} \left( \frac{x}{\sqrt{a}} \right) + f(x) \right\} \right] \right]_0^{(1-\delta)} \quad (4.13)$$

where  $f(x)$  is a function of order unity which we present in the appendix, and the symbol  $\zeta$  stands for

$$\zeta = 3.17\pi ne^2 b^2 / (\epsilon_0 - 1)m\gamma^2 v^2. \quad (4.14)$$

When putting in the limits of integration in the above expression, we find that the lower limit gives a zero contribution. At the upper limit since from (3.20) the quantity  $\delta$  is of order  $\eta^3$ , we will evaluate it as an expansion in powers of  $\delta$ . To this end we rearrange the above expression as

$$\frac{dU}{dz} = \frac{4\pi n(ee')^2}{(\epsilon_0 - 1)\beta^2 mc^2} \left\{ \left[ - \frac{\beta^2(\epsilon_0 - 1)}{2} \left( \log(1-x^2) + \frac{\eta x}{\pi(1-x)} \log \frac{\zeta x^2(a+x^2)}{(1-x^2)} \right) - \frac{x^2}{2\gamma^2} \right] \right. \\ \left. - \frac{\beta^2(\epsilon_0 - 1)}{2\pi} \eta \left[ \frac{x}{1+x} \log \frac{\zeta x^2(a+x^2)}{(1-x^2)} + \log(1-x) \log \left( \frac{1+x}{4} \right) \right. \right. \\ \left. \left. - 4 \left( \frac{a+2}{a+1} + \frac{1}{4} \log \frac{(a+1)\zeta^2 x^2}{(1-x^2)} \right) \tanh^{-1} x \right. \right. \\ \left. \left. + \frac{4\sqrt{a}}{a+1} \tan^{-1} \frac{x}{\sqrt{a}} + 2f(x) \right] \right\}_0^{(1-\delta)}. \quad (4.15)$$

In this form the dominant terms will be those in the first square bracket. The others will be of the order  $\eta$  or even smaller. If now we follow Fermi and compare in the final result only terms to zero order in  $\eta$ , then we need keep only the terms in the first square bracket above. Putting in the expression (3.19) for  $\delta$ , we obtain the

following final result for the energy loss to this order of magnitude:

$$\frac{dU}{dz} = \frac{4\pi n(ee')^2}{mc^2} \left[ -\frac{(\epsilon_0 - 1)}{4\pi\eta} \gamma^2 \log \left( \frac{3.17\pi(\epsilon_0 - 1)ne^2b^2}{4\eta^2 mc^2} \gamma^2 \right) - \frac{1}{2} \log \left( \frac{4\eta^2}{(\epsilon_0 - 1)} \frac{1}{\gamma^2} \right) - \frac{1}{2(\epsilon_0 - 1)} \frac{1}{\beta^2 \gamma^2} \right]. \tag{4.16}$$

We now make use of the characteristic parameters which we have introduced above in order to write this result in the following fashion:

$$\frac{dU}{dz} = \alpha' u_0 \left[ \frac{(\epsilon_0 - 1)}{4\pi\eta} \gamma^2 \log \left( \frac{16}{3.17(\epsilon_0 - 1)} \frac{\eta^2}{(b/\lambda_p)^2} \frac{1}{\gamma^2} \right) + \frac{1}{2} \log \left( \frac{(\epsilon_0 - 1)}{4\eta^2} \gamma^2 \right) - \frac{1}{2(\epsilon_0 - 1)} \frac{1}{\beta^2 \gamma^2} \right]. \tag{4.17}$$

We will again leave further consideration of this result to § 5 except to quote here the corresponding result of Tompkin (1965), which is used by Ahlen (1976):

$$\frac{dU}{dz} = \alpha' u_0 \left[ \frac{1}{2} \log \left( \frac{4}{3.17} \frac{1}{(b/\lambda_p)^2} \right) - \frac{1}{2} \frac{1}{(\epsilon_0 - 1)\beta^2 \gamma^2} \right]. \tag{4.18}$$

#### 4.2. Energy loss by a moving electric charge: $v > c'(\omega)$

Our starting point now will be (2.13). After applying to it the same considerations which we have just used in treating (2.10), we obtain the expression

$$\frac{dU}{dz} = \frac{4\pi ne^4}{(\epsilon_0 - 1)\beta^2 mc^2} \operatorname{Re} \int_0^{(1-\delta)} dx \left\{ x \left( \frac{(\epsilon_0 - 1)}{\epsilon_0 - x^2 - 2i\eta x} - \frac{1}{\gamma^2} \right) \times \left[ 1 + \frac{i}{\pi} \left( \log \left( \frac{3.17\pi ne^2 b^2}{(\epsilon_0 - 1) mc^2 \beta^2 \gamma^2} \right) + \log x^2 + \log \frac{a + x^2 + 2i\eta x}{1 - x^2 - 2i\eta x} \right) \right] \right\} \tag{4.19}$$

where  $a$  is given by (4.11). Just as in the preceding case of a magnetic charge we expand the above integral in powers of  $\eta$  and keep terms only to the first order in  $\eta$ . The result is as follows:

$$\frac{dU}{dz} = \frac{4\pi ne^4}{(\epsilon_0 - 1)\beta^2 mc^2} \int_0^{(1-\delta)} dx \left\{ \left( \frac{(\epsilon_0 - 1)x}{(\epsilon_0 - x^2)} - \frac{x}{\gamma^2} \right) + \frac{2}{\pi} \eta \left[ \frac{1}{\gamma^2} \left( \frac{x^2}{a + x^2} + \frac{x^2}{1 - x^2} \right) - (\epsilon_0 - 1) \left( \log \zeta \frac{x^2}{(\epsilon_0 - x^2)^2} + \frac{x^2}{(\epsilon_0 - x^2)} \left( \frac{1}{(a + x^2)} + \frac{1}{(1 - x^2)} \right) + \frac{x^2 \log x^2}{(\epsilon_0 - x^2)^2} + \frac{x^2}{(\epsilon_0 - x^2)^2} \log \left( \frac{a + x^2}{1 - x^2} \right) \right] \right\}. \tag{4.20}$$

We have written this result in this fashion so as to facilitate its comparison with the corresponding expression for the magnetic charge; namely, (4.12). The integrals in these two expressions are similar. The result of performing the above integration is

as follows:

$$\begin{aligned} \frac{dU}{dz} = & \frac{4\pi n e^4}{(\varepsilon_0 - 1)\beta^2 m c^2} \left\{ - \left( \frac{\varepsilon_0 - 1}{2} \log(\varepsilon_0 - x^2) + \frac{x^2}{2\gamma^2} \right) \right. \\ & - \frac{\beta^2(\varepsilon_0 - 1)}{\pi} \eta \left[ \frac{x}{(\varepsilon_0 - x^2)} \log \left( \frac{\zeta x^2(a + x^2)}{(1 - x^2)} \right) \right. \\ & - \frac{2}{\sqrt{\varepsilon_0}} \left( 1 + \frac{1}{2} \log \left( \frac{\zeta(\varepsilon_0 + a)x^2}{(1 - x^2)} \right) \right) \tanh^{-1} \frac{x}{\sqrt{\varepsilon_0}} \\ & + \frac{2}{\sqrt{\varepsilon_0}} \left( \tanh^{-1} \frac{1}{\sqrt{\varepsilon_0}} - \frac{\sqrt{\varepsilon_0}}{(\varepsilon_0 - 1)\gamma^2} \right) \tanh^{-1} x \\ & \left. \left. + \frac{2}{\sqrt{a}} \left( \varepsilon_0 - \frac{(\varepsilon_0 + 1)}{(\varepsilon_0 - 1)\gamma^2} \right) \tan^{-1} \frac{x}{\sqrt{a}} + g(x) \right] \right\}^{(1-\delta)} \end{aligned} \quad (4.21)$$

where  $g(x)$  is a function of order unity which we list in the appendix, together with the result of evaluating the last two integrals in (4.20).

In the same way as we did with (4.13), we rearrange the terms in the above equation so as to group the dominant terms in the first bracket and the terms of order  $\eta$  or higher in the second bracket. The result is as follows:

$$\begin{aligned} \frac{dU}{dz} = & \frac{4\pi n e^4}{(\varepsilon_0 - 1)\beta^2 m c^2} \left\{ \left[ - \frac{\varepsilon_0 - 1}{2} \left( \log(\varepsilon_0 - x^2) \right. \right. \right. \\ & + \left. \left. \frac{\beta^2 \eta}{\eta \sqrt{\varepsilon_0}} \frac{x}{(\sqrt{\varepsilon_0} - x)} \log \frac{\zeta x^2(a + x^2)}{(1 - x^2)} - \frac{x^2}{2\gamma^2} \right] \right. \\ & - \frac{\beta^2(\varepsilon_0 - 1)}{\pi} \eta \left[ \frac{1}{2\sqrt{\varepsilon_0}} \frac{x}{(\sqrt{\varepsilon_0} + x)} \log \frac{\zeta x^2(a + x^2)}{(1 - x^2)} \right. \\ & - \frac{2}{\sqrt{\varepsilon_0}} \left( 1 + \frac{1}{2} \log \frac{\zeta(\varepsilon_0 + a)x^2}{(1 - x^2)} \right) \tanh^{-1} \frac{x}{\sqrt{\varepsilon_0}} \\ & + \frac{2}{\sqrt{\varepsilon_0}} \left( \tanh^{-1} \frac{1}{\sqrt{\varepsilon_0}} - \frac{\sqrt{\varepsilon_0}}{(\varepsilon_0 - 1)\gamma^2} \right) \tanh^{-1} x \\ & \left. \left. + \frac{2}{\sqrt{a}} \left( \varepsilon_0 - \frac{\varepsilon_0 - 1}{\varepsilon_0 - 1} \frac{1}{\gamma^2} \right) \tan^{-1} \frac{x}{\sqrt{a}} + g(x) \right] \right\}^{(1-\delta)}. \end{aligned} \quad (4.22)$$

Just as in the preceding case, we evaluate this expression by neglecting all terms of the first or higher order in  $\eta$ . We obtain the following final result:

$$\begin{aligned} \frac{dU}{dz} = & \frac{4\pi n e^4}{m c^2} \left[ \frac{1}{\beta^2} \log \frac{1}{(\varepsilon_0 - 1)^{1/2}} - \frac{1}{2\pi\sqrt{\varepsilon_0}} \frac{\eta}{(\sqrt{\varepsilon_0} - 1)} \log \left( \frac{3.17\pi (\varepsilon_0 - 1) n e^2 b^2}{4\eta^2 m c^2} \gamma^2 \right) \right. \\ & \left. - \frac{1}{2(\varepsilon_0 - 1)} \frac{1}{\beta^2 \gamma^2} \right]. \end{aligned} \quad (4.23)$$

It would be pertinent for us at this stage to make the following remark concerning the second term in (4.23) above. Due to the factor  $\eta$  which occurs in it, one might

think that it is of first order in  $\eta$  and hence must be neglected. However, as we will show in § 5 the quantity  $(\sqrt{\epsilon_0} - 1)$  which occurs in the denominator is of order  $\sqrt{\eta}$ . Hence, the whole term is of order  $\sqrt{\eta}$ .

We rewrite (4.23) as follows:

$$\frac{dU}{dz} = \alpha u_0 \left[ \frac{1}{2\pi\sqrt{\epsilon_0}} \frac{\eta}{(\sqrt{\epsilon_0} - 1)} \log \left( \frac{16}{3.17(\epsilon_0 - 1)} \frac{\eta^2}{(b/\lambda_p)^2 \gamma^2} \right) + \left( \log \frac{1}{(\epsilon_0 - 1)^{1/2}} \right) \frac{1}{\beta^2} - \frac{1}{2(\epsilon_0 - 1)} \frac{1}{\beta^2 \gamma^2} \right]. \tag{4.24}$$

The following is the corresponding result as obtained by Fermi:

$$\frac{dU}{dz} = \alpha u_0 \left[ \frac{1}{2\beta^2} \log \left( \frac{4}{3.17} \left( \frac{\lambda_p}{b} \right)^2 \beta^2 \right) - \frac{1}{2(\epsilon_0 - 1)} \frac{1}{\beta^2 \gamma^2} \right]. \tag{4.25}$$

### 5. Comparison with Fermi's and Ahlen's results

In order to facilitate the comparison of our results with those of Fermi and Ahlen, we follow Ahlen (1980) and introduce two dimensionless functions  $L$  and  $L'$  to describe the energy loss of electric and magnetic charges, respectively:

$$dU/dz = \alpha u_0 L, \quad dU/dz = \alpha' u_0 L'. \tag{5.1}$$

These functions will depend on  $\beta^2$  and on  $b$ , the minimum impact parameter. However, we must now choose an expression for  $b$  so as to exhibit the dependence of the energy loss on  $\beta^2$ . The natural choice for us is that made by Ahlen (1976); namely,

$$b = \lambda_e / \beta \gamma \tag{5.2}$$

where  $\lambda_e$  is the Compton wavelength of the electron. The above expression for  $b$  is equal to the de Broglie wavelength of the electron in the CM frame for its scattering with a heavy incident particle. Ahlen (1976) shows that expressions for the energy loss of the type which we present here are appropriate to the description of track characteristics of charged particles in plastic detectors. To obtain a good approximation to the total energy loss he recommends the above choice for  $b$ . The resulting expressions can then be construed as giving a meaningful comparison between the behaviour of electric and magnetic charges in matter.

Putting (5.2) in (4.17) and (4.18) for the energy loss by a magnetic charge, we obtain

$$\frac{dU}{dz} = \alpha' u_0 \left[ \frac{(\epsilon_0 - 1)}{4\pi\eta} \gamma^2 \log \left( \frac{16\eta^2 (\lambda_p/\lambda_e)^2}{3.17(\epsilon_0 - 1)} \beta^2 \right) + \frac{1}{2} \log \left( \frac{(\epsilon_0 - 1)}{4\eta^2} \gamma^2 \right) - \frac{1}{2(\epsilon_0 - 1)} \frac{1}{\beta^2 \gamma^2} \right], \tag{5.3}$$

$$\frac{dU}{dz} = \alpha' u_0 \left[ \frac{1}{2} \log \left( \frac{4(\lambda_p/\lambda_e)^2}{3.17} \beta^2 \gamma^2 \right) - \frac{1}{2(\epsilon_0 - 1)} \frac{1}{\beta^2 \gamma^2} \right]. \tag{5.4}$$

Equation (5.4) represents the energy loss expression used by Ahlen. We will soon show that these expressions hold for ultrarelativistic velocities. For such velocities we see that our expression for the energy loss diverges like  $\gamma^2$ :

$$\frac{dU}{dz} \underset{\beta \rightarrow 1}{\sim} \left[ \alpha' u_0 \frac{\epsilon_0 - 1}{4\pi\eta} \log \left( \frac{16\eta^2 (\lambda_p/\lambda_e)^2}{3.17(\sigma - 1)} \right) \right] \gamma^2. \tag{5.5}$$



However, the expression used by Ahlen diverges like  $\log \gamma^2$ :

$$\left(\frac{dU}{dz}\right)_{\text{Ahlen}} \underset{\beta \rightarrow 1}{\sim} \frac{1}{2} \alpha' u_0 \log \gamma^2. \quad (5.6)$$

It will be seen that the above behaviour for (5.3) comes from the first term and is directly related to the upper limit of integration in (4.11).

The corresponding expressions for the energy loss by an electric charge; namely, (4.24) and (4.25), become

$$\begin{aligned} \frac{dU}{dz} = \alpha u_0 \left[ \frac{\eta}{2\pi\sqrt{\epsilon_0}(\sqrt{\epsilon_0}-1)} \log \left( \frac{16\eta^2(\lambda_p/\lambda_e)^2}{3.17(\epsilon_0-1)} \beta^2 \right) \right. \\ \left. + \log \left( \frac{1}{(\epsilon_0-1)^{1/2}} \right) \frac{1}{\beta^2} - \frac{1}{2(\epsilon_0-1)} \frac{1}{\beta^2 \gamma^2} \right], \end{aligned} \quad (5.7)$$

$$\left(\frac{dU}{dz}\right)_{\text{Fermi}} = \alpha u_0 \left[ \frac{1}{2\beta^2} \log \left( \frac{4(\lambda_p/\lambda_e)^2}{3.17} \beta^4 \gamma^2 \right) - \frac{1}{2(\epsilon_0-1)} \frac{1}{\beta^2 \gamma^2} \right]. \quad (5.8)$$

Again the behaviour at ultrarelativistic velocities is quite distinct. Our calculation gives a finite result; namely

$$\frac{dU}{dz} \underset{\beta \rightarrow 1}{\sim} \alpha u_0 \frac{\eta}{2\pi\sqrt{\epsilon_0}(\sqrt{\epsilon_0}-1)} \log \left( \frac{16\eta^2(\lambda_p/\lambda_e)^2}{3.17(\epsilon_0-1)} \right). \quad (5.9)$$

Fermi's calculation leads to the same kind of logarithmic divergence as in the Ahlen result above:

$$\left(\frac{dU}{dz}\right)_{\text{Fermi}} \underset{\beta \rightarrow 1}{\sim} \frac{1}{2} \alpha u_0 \log \gamma^2. \quad (5.10)$$

Again the above behaviour of our result for the energy loss is traceable to the upper limit of integration in (4.19).

This serves to highlight the dependence of the calculation of the energy loss on the model used for the dielectric constant. A proper calculation should use a more realistic model. However, the present results should serve to illustrate the importance of handling the various ranges of velocity in the proper fashion.

### 5.1. Numerical results

In order to be able to put our calculations in numerical form we must fix the parameters which occur in our model for the dielectric constant. We will choose water as our medium, which will then fix  $n$  to be equal to  $3.35 \times 10^{23} \text{ cm}^{-3}$  and will give  $\omega_p = 3.29 \times 10^{16} \text{ s}^{-1}$ . As for  $\omega_0$  we will adopt for it the value determined by the ionisation potential for water which has been used by Sternheimer (1956) in his calculation of the density effect; namely  $I = 74.1 \text{ eV}$ . This gives  $\omega_0 = 1.12 \times 10^{17} \text{ s}^{-1}$ . Using these values in (3.4) we obtain  $\epsilon_0 = 1.09$ . For the justification of this procedure we refer to Ahlen (1976).

We turn now to the evaluation of  $\eta$ . From Jackson (1975) we have the following expression for  $\eta$ :

$$\eta = \frac{1}{2} \frac{\epsilon_0^{1/2}}{(\epsilon_0-1)^{3/2}} \left(\frac{\omega_p}{\omega}\right)^2 \lambda_p \alpha(\omega) \quad (5.11)$$

where  $\alpha(\omega)$  is the absorption coefficient for liquid water. We must choose a value for  $\alpha(\omega)$  which is consistent with our assumption of weak absorption. By comparison with figure 7.9 in Jackson (1975), we find that the choice  $\alpha(\omega) \approx 10^{-1} \text{ cm}^{-1}$  is not too excessive in either direction. This choice fixes the value of  $\omega$  in the preceding equations to be  $3.98 \times 10^{14} \text{ s}^{-1}$ . Putting all these results in (5.1) we obtain

$$\eta = 1.30 \times 10^{-2}. \quad (5.12)$$

This value of  $\eta$  gives *a posteriori* justification for our choice of parameters. It is seen to correspond to weak dissipation.

Putting the values of  $\epsilon_0$  and  $\eta$  into the inequality (3.17), we find that the range of validity of our results; namely, (4.17) and (4.24), is given by

$$0.917 < \beta^2 < 0.987. \quad (5.13)$$

These expressions are then supposed to hold in the ultrarelativistic domain of velocities.

We now put the above numerical values for the parameters in (5.3) and (5.4) to obtain the following expressions for the energy loss by a magnetic charge:

$$\frac{dU}{dz} = \alpha' u_0 [0.551 \gamma^2 \log(6.21 \times 10^6 \beta^2) + \frac{1}{2} \log(1.33 \times 10^2 \gamma^2) - 5.56/\beta^2 \gamma^2], \quad (5.3a)$$

$$(dU/dz)_{\text{Ahlen}} = \alpha' u_0 [\frac{1}{2} \log(8.26 \times 10^8 \beta^2 \gamma^2) - 5.56/\beta^2 \gamma^2]. \quad (5.4a)$$

In figure 2 we plot the dimensionless functions  $L'$  and  $L'_{\text{Ahlen}}$  as functions of  $\beta^2$  to give an idea of their relative orders of magnitude.

With the same substitution, equations (5.7) and (5.8) for the energy loss of an electric charge acquire the following forms:

$$dU/dz = \alpha u_0 [0.0498 \log(6.21 \times 10^6 \beta^2) + 1.20/\beta^2 - 5.56/\beta^2 \gamma^2], \quad (5.7a)$$

$$(dU/dz)_{\text{Fermi}} = \alpha u_0 [(2\beta^2)^{-1} \log(8.26 \times 10^8 \beta^4 \gamma^2) - 5.56/\beta^2 \gamma^2]. \quad (5.8a)$$

We show the corresponding dimensionless functions in figure 3.

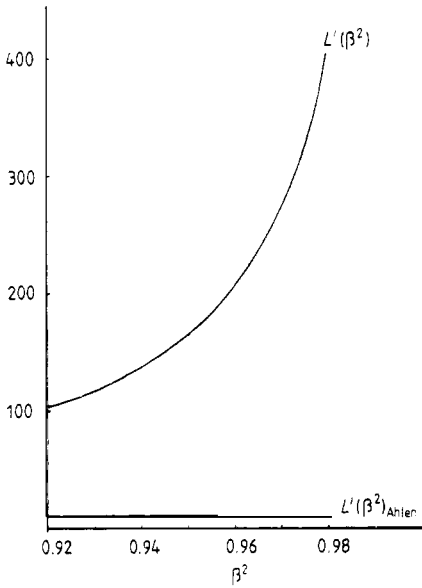
Figures 2 and 3 give a clear indication of the sizable modification which our method brings about in the energy loss. Our result for magnetic monopoles is seen to be appreciably higher than that given by Ahlen in the whole velocity range. It diverges strongly from it towards high values of the velocity. For electric charges our result is consistently lower than Fermi's for the whole velocity range.

## 6. On the detection of dyons

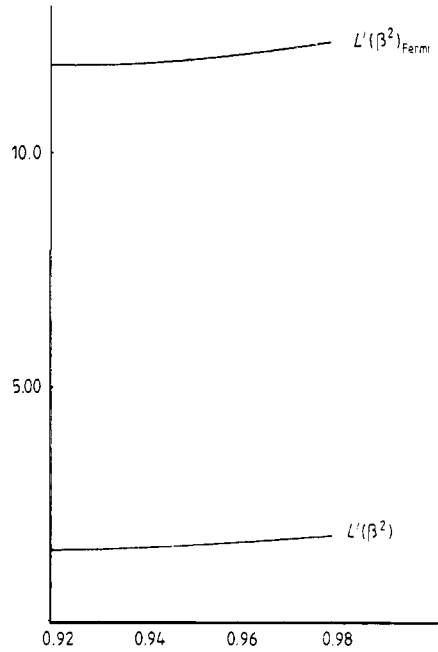
For a dyon carrying electric charge  $e$  and magnetic charge  $e'$  the fields will be given by the sum of the respective fields which we gave in § 2. Due to the form of these fields there will be no cross terms in the Poynting flux coming from the product of the fields of the electric charge with those of the magnetic charge. The energy loss for a dyon will then be given by

$$dU/dz = u_0 [\alpha L(\beta^2) + \alpha' L'(\beta^2)] \quad (6.1)$$

where  $L(\beta^2)$  and  $L'(\beta^2)$  are the dimensionless functions which we introduced in § 5. From this and our preceding results we see that the energy loss by a dyon is higher than that for either an electric charge or a magnetic monopole for all velocities. At



**Figure 2.** The dimensionless energy loss function for an ultrarelativistic magnetic charge.  $L'(\beta^2)$  is our result and  $L'(\beta^2)_{\text{Ahlen}}$  is that of Ahlen.



**Figure 3.** The dimensionless energy loss function for an ultrarelativistic electric charge.  $L'(\beta^2)$  is our result and  $L'(\beta^2)_{\text{Fermi}}$  is Fermi's.

ultrarelativistic velocities it will be characterised by a monopole-type behaviour, i.e. energy loss diverging as  $\gamma^2$ . For low velocities it will show the dominant behaviour of an electric charge; namely, energy loss varying as  $1/\beta^2$ .

To be more specific than this, a proper treatment of the energy loss must be undertaken. This involves using the proper expression for the dielectric constant and then repeating our calculation of  $L$  and  $L'$  for the whole velocity range. The resulting expression for the dyon should give a good description for track characteristics in plastic detectors (Ahlen 1976). This will be in the form of a family of curves characterised by the unknown parameter  $(e'/e)^2$ . This parameter would, of course, be determined by a detection of a dyon by an experiment utilising energy loss by ionisation along the lines outlined here.

6.1. Determination of the relative charge of the dyon

The method which we have just suggested for the detection of the dyon can only measure the magnitude of its charges. To determine the relative sign of these charges we must seek some other method. We suggest here a method which would work for dyons which move fast enough so as to emit Cerenkov radiation. This consists in measuring the polarisation of the emitted radiation. We will now explain this method which must be considered as complementary to the method of energy loss which we have sketched above.

We start from the fields of § 2 for the case  $v > c'(\omega)$ . Taking the asymptotic form of the fields and ignoring dissipation, we can readily show that the fields of a dyon

are given as follows:

$$\begin{aligned}
 \mathbf{E} &= \frac{2}{(2\pi)^{1/2}} (\beta c^3 \rho)^{-1/2} \int_{\kappa^2(\omega) > 1/\beta^2} d\omega \frac{I_1(\omega)}{(\epsilon(\omega))^{1/2}} (\mu(\omega)e^2 + \epsilon(\omega)e'^2)^{1/2} \left(\frac{\omega}{\gamma'(\omega)}\right)^{1/2} \\
 &\quad \times \cos \left[ \omega \left( \frac{\mathbf{n}(\omega) \cdot \mathbf{r}}{c'(\omega)} - t \right) - \frac{\pi}{4} \right], \\
 \mathbf{H} &= \frac{2}{(2\pi)^{1/2}} (\beta c^3 \rho)^{-1/2} \int_{\kappa^2(\omega) > 1/\beta^2} d\omega \frac{I_2(\omega)}{(\mu(\omega))^{1/2}} (\mu(\omega)e^2 + \epsilon(\omega)e'^2)^{1/2} \left(\frac{\omega}{\gamma'(\omega)}\right)^{1/2} \\
 &\quad \times \cos \left[ \omega \left( \frac{\mathbf{n}(\omega) \cdot \mathbf{r}}{c'(\omega)} - t \right) - \frac{\pi}{4} \right].
 \end{aligned} \tag{6.2}$$

In the above expressions all symbols have been defined above except for the vectors. We now explain how they are defined. The vector  $\mathbf{n}(\omega)$  is a unit vector which we define as

$$\mathbf{n}(\omega) = (\gamma'(\omega)\boldsymbol{\rho}_1 - \mathbf{k})/\beta'(\omega)\gamma'(\omega). \tag{6.3}$$

It forms a complete basis together with  $\boldsymbol{\varphi}_1$  and the unit vector  $\mathbf{l}(\omega)$  which we define as

$$\mathbf{l}(\omega) = \boldsymbol{\varphi}_1 \times \mathbf{n}(\omega). \tag{6.4}$$

Figure 4 illustrates the meaning of these vectors.  $\mathbf{l}(\omega)$  generates the Cerenkov cone which envelops the Cerenkov radiation with frequency  $\omega$ . The vector  $\mathbf{n}(\omega)$  is normal to this cone and points along the direction of the Poynting flux.

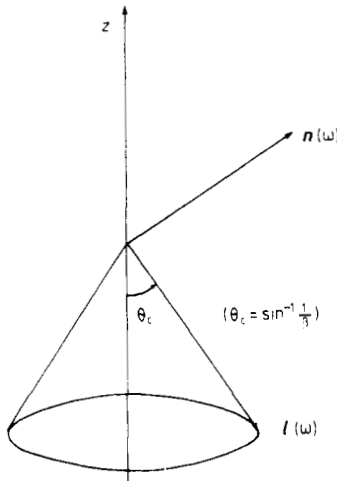


Figure 4. The Cerenkov cone and the vectors  $\mathbf{l}(\omega)$  and  $\mathbf{n}(\omega)$  and the Cerenkov angle  $\theta_c$ .

To define the polarisation vector in (6.2) we need to introduce an angle  $\theta_d(\omega)$  which we will define as follows:

$$\theta_d(\omega) = \tan^{-1}(\epsilon(\omega)/(\mu(\omega))^{1/2}(e'/e)). \tag{6.5}$$

We recall that the susceptibilities are now real since we are ignoring dissipation. The polarisation vector  $\mathbf{l}_1(\omega)$  is then given by

$$\mathbf{l}_1(\omega) = (\cos \theta_d(\omega)\mathbf{l}(\omega) + \sin \theta_d(\omega)\boldsymbol{\varphi}_1). \tag{6.6}$$

The vector  $I_2(\omega)$  is given by

$$I_2(\omega) = \mathbf{n} \times I_1(\omega) \tag{6.7}$$

which means that also in the case of the dyon the energy flows normal to the Cerenkov cone.

Equation (6.6) shows that a measurement of the polarisation of the Cerenkov radiation is central to the determination of the type of particle emitting this radiation. For an electric charge  $\theta_a(\omega) = 0$ , and for a magnetic charge it equals  $\pi/2$ . Any other value of this angle would mean the existence of a dyon and would give its relative charge as seen from (6.5). Coupled with the method of energy dissipation, this would give the value of either charge in terms of the other.

### 7. Conclusions

We have argued above that a proper calculation of the density effect should start with the exact expressions for the electromagnetic fields of a charged particle in matter. The expressions for the susceptibilities which are used in these equations would then result in restrictions which must be handled correctly. We have demonstrated this within the ambit of the Fermi model. The lack of a proper treatment of the conditions imposed by the model leads to an appreciable variation in the results.

We feel justified in concluding that a proper calculation using the correct expressions for the susceptibilities would be valuable and should be compared with experiment. Such a result for plastic detectors, for instance, should be useful in establishing track characteristics for magnetic monopoles and for dyons.

### Appendix. Some integrals and functions

#### A.1. The last two integrals in (4.12)

$$(i) \int \frac{x^2 \log x^2}{(1-x^2)^2} dx = \left( \frac{x \log x}{(1-x^2)} - (1 + \log x) \tanh^{-1} x + \sum_{n=0}^{\infty} \frac{x^{(2n+1)}}{(2n+1)^2} \right).$$

$$(ii) \int \frac{x^2}{(1-x^2)^2} \log \left( \frac{a+x^2}{1-x^2} \right) dx$$

$$= \left\{ \frac{x}{2(1-x^2)} \left[ \log \left( \frac{a+x^2}{1-x^2} \right) - 1 \right] + \frac{1}{4} \log(1-x) \log \left( \frac{1+x}{4} \right) \right.$$

$$+ \frac{1}{2} \left[ \left( \frac{a-1}{a+1} \right) - \frac{1}{2} \log \left( \frac{a+1}{1-x^2} \right) \right] \tanh^{-1} x + \frac{\sqrt{a}}{(a+1)} \tan^{-1} \frac{x}{\sqrt{a}}$$

$$+ \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left( \frac{1-x}{2} \right)^n - \frac{1}{4} \sum_{n=1}^{\infty} \left[ \frac{i^n}{n^2} \left( \frac{1-x}{\sqrt{a+i}} \right)^n + \text{cc} \right]$$

$$\left. + \frac{1}{4} \sum_{n=1}^{\infty} \left[ \frac{i^n}{n^2} \left( \frac{1+x}{\sqrt{a+i}} \right)^n + \text{cc} \right] \right\}.$$

A.2. The function  $f(x)$  in (4.13)

$$f(x) = \left\{ \frac{1}{2} \sum_{n=1}^{\infty} \left[ \frac{i^n}{n^2} \left( \frac{1+x}{\sqrt{a+i}} \right)^n + \text{CC} \right] - \frac{1}{2} \sum_{n=1}^{\infty} \left[ \frac{i^n}{n^2} \left( \frac{1-x}{\sqrt{a+i}} \right)^n + \text{CC} \right] \right. \\ \left. + \sum_{n=1}^{\infty} \frac{1}{n^2} \left( \frac{1-x}{2} \right)^n + 2 \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)^2} \right\}.$$

A.3. The last two integrals in (4.20)

$$(i) \int \frac{x^2 \log x^2}{(\epsilon_0 - x^2)^2} dx = \left[ \frac{x \log x}{(\epsilon_0 - x^2)} - \frac{1}{\sqrt{\epsilon_0}} (1 + \log x) \tanh^{-1} \frac{x}{\sqrt{\epsilon_0}} \right. \\ \left. + \frac{1}{\sqrt{\epsilon_0}} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \left( \frac{x}{\sqrt{\epsilon_0}} \right)^{(2n+1)} \right].$$

$$(ii) \int \frac{x^2}{(\epsilon_0 - x^2)^2} \log \left( \frac{a+x^2}{1-x^2} \right) dx = \left[ \frac{x}{2(\epsilon_0 - x^2)} \log \left( \frac{a+x^2}{1-x^2} \right) \right. \\ \left. + \frac{1}{\sqrt{\epsilon_0}} \left[ \frac{\epsilon_0(a+1)}{(\epsilon_0-1)(\epsilon_0+a)} - \frac{1}{2} \log \left( \frac{\epsilon_0+a}{1-x^2} \right) \right] \tanh^{-1} \left( \frac{x}{\sqrt{\epsilon_0}} \right) \right. \\ \left. + \frac{1}{\sqrt{\epsilon_0}} \left( \tanh^{-1} \frac{1}{\sqrt{\epsilon_0}} - \frac{\sqrt{\epsilon_0}}{(\epsilon_0-1)} \right) \tanh^{-1} x + \frac{\sqrt{a}}{(\epsilon_0+a)} \tan^{-1} \frac{x}{\sqrt{a}} \right. \\ \left. + \frac{1}{\sqrt{\epsilon_0}} \left[ \frac{1}{4} \sum_{n=1}^{\infty} \frac{1}{n^2} \left[ \left( \frac{1-x}{\sqrt{\epsilon_0+1}} \right)^n - \left( \frac{1+x}{\sqrt{\epsilon_0+1}} \right)^n \right] \right. \right. \\ \left. \left. + \frac{1}{4} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left[ \left( \frac{1+x}{\sqrt{\epsilon_0-1}} \right)^n - \left( \frac{1-x}{\sqrt{\epsilon_0-1}} \right)^n \right] \right. \right. \\ \left. \left. - \frac{1}{4} \sum_{n=1}^{\infty} \left[ \frac{i^n}{n^2} \left( \frac{\sqrt{\epsilon_0-x}}{\sqrt{a+i\sqrt{\epsilon_0}}} \right)^n + \text{CC} \right] + \frac{1}{4} \sum_{n=1}^{\infty} \left[ \frac{i^n}{n^2} \left( \frac{\sqrt{\epsilon_0+x}}{\sqrt{a+i\sqrt{\epsilon_0}}} \right)^n + \text{CC} \right] \right] \right].$$

A.4. The function  $g(x)$  in (4.21)

$$g(x) = \frac{1}{2\sqrt{\epsilon_0}} \left\{ \sum_{n=1}^{\infty} \frac{1}{n^2} \left[ \left( \frac{1-x}{\sqrt{\epsilon_0+1}} \right)^n - \left( \frac{1+x}{\sqrt{\epsilon_0+1}} \right)^n \right] \right. \\ \left. + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left[ \left( \frac{1+x}{\sqrt{\epsilon_0-1}} \right)^n - \left( \frac{1-x}{\sqrt{\epsilon_0-1}} \right)^n \right] \right. \\ \left. - \sum_{n=1}^{\infty} \left[ \frac{i^n}{n^2} \left( \frac{\sqrt{\epsilon_0-x}}{\sqrt{a+i\sqrt{\epsilon_0}}} \right)^n + \text{CC} \right] + \sum_{n=1}^{\infty} \left[ \frac{i^n}{n^2} \left( \frac{\sqrt{\epsilon_0+x}}{\sqrt{a+i\sqrt{\epsilon_0}}} \right)^n + \text{CC} \right] \right\}.$$

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